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INTRODUCTION

TO THE

MATHEMATICAL PRINCIPLES OF THE

NEBULAR THEORY, OR PLANETOLOGY.

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The *nebular hypothesis*—the boldest thought that ever elevated the human mind, by bringing us, as it were, in sight of the mysterious fiat of the Almighty—was, in its great general features, unfolded almost at the same time by Germany's deepest thinker, the Königsberg philosopher, Immanuel Kant, and by Pierre Simon de Laplace, the greatest mathematician of France. It is truly the closing stone in the philosophy of the celestial vault; for Copernicus and Kepler made us behold the foundation,—the first, by placing the sun as the lantern of the world in the center, and surrounding it with the planets—the second, by destroying the cycles and unravelling the harmony of the spheres in his immortal laws; and after the existing phenomena had thus been rightly viewed, Newton made us behold the invisible bond that connects the members of the system, while at length Kant and Laplace pointed out to us the hand that at "the beginning" projected these celestial balls into space and thereby insured the continued existence of the system.

But notwithstanding this noble parentage and its being the *logical sequence* of the discoveries in the theory of cosmos made by Copernicus, Kepler and Newton, the nebular theory enjoys as yet but slight consideration among astronomers. Arago[1] is the only one of these who has deigned to consider it earnestly, and he probably did so more in his capacity as a physicist than as an astronomer.

[1] Arago, Astronomie Populaire, ii, 7. Paris and Leipsic, 1855.

The reason of this neglect seems to be the incomplete state in which even Laplace himself left the theory. Direct observation, moreover, seemed to contradict some laws given as necessary consequences of this hypothesis.

We have already, in a former article,[2] tried to vindicate the theory in this last respect by showing that the hypothesis is really confirmed even in these apparently contradicting observations. We will now endeavor to give a somewhat more complete development to the fundamental principles of Kant and Laplace, and to exhibit the exact position of the nebular theory itself, hoping thereby to show that this theory, if we only study it earnestly and patiently both by experiment and analysis, fully deserves our confidence.

As this subject is as vast as it is difficult, we beg the critic always to keep in mind that we do not pretend to give a treatise, but merely offer an *introduction* to this almost new field of analysis.

We commence with a short survey of the fundamental principles and the aim of the theory of the solar system, in order clearly to understand why the nebular theory is necessary, what it will have to accomplish, and how far it already has done its duty.

§ 1. *The fundamental constants of the Solar system.*

As the discovery of a law of nature is but the reduction of the infinitude of observed quantities to a few constants by means of a function, the algebraic expression of the law—we see that *the progress of astronomy to a great extent must be identical with the reduction of the number of such constants.* This is fully borne out by the history of the science. For, while the Ptolemaic theory[3] of the planetary motion required the radii and inclinations of seventeen different circles to express the observed motions of Saturn, Kepler reduced this number of constants to three, the semi-major axis, eccentricity, and inclination of the orbit. This very principle is also placed by Laplace[4] at the head of his *Mécanique Céléste.*

We may therefore trust in this principle, and with Laplace try to reduce the number of indispensable constants. We must first, however, ascertain which are those constants that are now *considered* fundamental or indispensable.

The *constants of the solar system* now exclusively deduced from observation are:

1. The *mass*, m, of the planet.

[2] The density, rotation, and relative age of the planets: this Journal, Jan., 1864.

[3] Fracastor; see Bailly, Histoire de l'Astronomie moderne; vol. i. Paris, 1779. Eclaircissements, livre iv, § 23, and livre viii, § 27.

[4] Il importe extrêmement d'en bannir tout empirisme et de la réduire à n'emprunter de l'observation que les données indispensables.—*Méc. Cél.—Plan.*

2. The *figure.* On assuming a primitive fluidity and a small velocity of rotation, theory gives an ellipsoidal figure of the planets; hence not independent of observation.

3. The *ellipticity* of the planet. Theory may assign to it a higher and a lower limit by means of the mass (1) and the angular velocity (6)—but though the connection of this constant with others thereby is manifest, still its exact value can only be derived from observation.

4. The *volume*, or *diameter*, of a planet. Combining this constant with the first (mass) we obtain the *density.*

5. The *plane* (or *inclination* of the axis),

6. The *direction*, and

7. The *velocity of rotation.* Though this last element bears relation to others, still its exact value can only be obtained from observation. Even if Kirkwood's law of rotation[6] should prove to be perfectly exact, this element would continue to be a fundamental constant as long as that law remains an empirical one.

8. The *distance*, a, of a planet from the sun, or the semi-major axis of its orbit. By the theoretically proved third law of Kepler, we get from this constant the *periodic time*, T, requiring only the *constant*, μ, *of gravitation* to be known, and this latter is the same for all planets.

9. The *plane* or *inclination*, i, of the orbit.

10. The *direction* of the motion. The *velocity* is given by the distance.

11. The *eccentricity of the orbit.* This constant *is* fundamental, for the theory of gravitation only proves the orbit to be a *conic section* of some kind. The eccentricity can only be found from observation.

12. The *number of satellites* of a planet is also fundamental—and for each of them the same eleven constants have to be taken from observation; the first seven even are required by the sun.

From these twelve empirical data, theoretical astronomy can deduce the motion of the corresponding planet. The whole number of empirical constants is not at all inconsiderable; for

8 principal planets,	constants	1–12,	96
80 small planets,	"	8–11,	320
23 secondary planets,	"	1–11,	253
The sun,	"	1–7,	7
Total number of constants,			676

to which the corresponding constants for the comets would have to be added.

What, in the face of this great number of constants that astronomy has to borrow from observation, shall we say about the boasted perfection of this science? Is it not in science, as in

[6] This Journal, ix, 395, May, 1850; also xiv, 210, Sept., 1852.

morals, that self-adoration hinders progress? Can any astronomer who has not merely studied the details of the celestial mechanics, but also kept in mind the great principle laid down by its author in his "Plan"—can he still pretend that Newton's theory of the solar system merely needs further development, seeing that the few bodies of this system require him to borrow about seven hundred constants from observation?

We shall honor the memory of Newton much more by trying to go beyond the results of his labors than by stupidly worshipping[6] the same, and thus arresting the progress of that science to promote which he spent his life.

§ 2. *These fundamental constants sustain remarkable relations to each other.*

The Newtonian theory of gravitation simply accepts these constants as observation gives them. For if our earth had Jupiter's mass, the rings of Saturn, the moons of Uranus and its axis in the ecliptic, the latter perpendicular to the orbit of Jupiter, a retrograde motion in a hyperbolic orbit—it still would as fully and as beautifully confirm the theory of universal gravitation as it does now; for, let us openly and frankly acknowledge it, these constants are independent of the theory of gravitation because the latter is independent of the former.

But, though this theory does not give any reasons for any kind of dependence between the often mentioned constants, observation shows that they sustain very remarkable relations to each other;[7] or in other words, *there are relations and laws in our solar*

[6] The literature of astronomy teems with implicit instances hereof; but we find also direct expressions of this feeling, like the following:

Enfin, nous avons vu que ces resultats eux mêmes peuvent se composer en un seul et se representer par une loi unique, celle de la Pésanteur universelle; parvenus à principe nous nous voyons en quelque sorte élévé à *la source commune de tous les faits astronomiques; tous en dérivent* de la manière la plus simple et ils y sont en quelque sorte comme concentrés. Nous avons donc pour ainsi dire décomposé le systéme du monde, *nous l'avons réduit à son élément unique*, et nous l'avons ensuite recomposé.—Biot, *Traité élém. d'astronomie physique.* Paris, 1805. Concluding remark of the work.

This "élément unique" is rather singularly unique, requiring no less than seven hundred elements to be borrowed from observation alone!

[7] Newton was aware of this—indeed, nobody can help seeing some of these relations. In the scholium to the third book of Principia he says:

"Planetæ sex principalis revolvuntur circum solem in circulis soli concentricis, eâdem motus directione, in eodem plano quamproximé. Lunæ decem revolvuntur circum terram, jovem et saturnam in circulis concentricis, eâdem motus directione, in planis orbium planetarum quamproximæ. *Et hi omnes motus regulares originem non habent ex causis mechanicis.*" Edit. *le Seur* et *Jacquier* Geneva 1749–42.

It is customary to censure Kepler's fancy in contrast to the solidity of all Newton's words; still a sentence like the above is much more objectionable in science than the boldest fancy, for the latter is not accepted without severe scrutiny, while the former is repeated as a sacred truth. If Newton had written "mechanical causes known to me" instead of by "mechanical causes," simply to imply that he knew them all, he would have prevented many a drawback that has encumbered

world of a still higher order than those deduced from gravitation. Thus, the inclinations of the orbits of the principal planets, instead of being uniformly distributed over the first quadrant, are all very small; and their direction, instead of being as often retrograde as direct, are for *all* planets and most satellites *direct.* Instead of having the eccentricities regularly varying from 0 to ∞, we find them for all planets nearly zero! The same may be said of any of the above fundamental constants, and not least of the distance, as it is found *approximating* to Titius–Bode's law, that is, to

$$a_n = 4 + 3 \cdot 2^{n-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But quantities that sustain mutual relations to each other are the particular values of a certain function for definite given values of the variable quantities; hence, if we intend to be true to the spirit expressed by the words of Laplace above quoted, it is a problem legitimately belonging to astronomy *to find these functions of which the fundamental constants are but particular values.*

Let us boldly face this great problem and not desist though astronomers tell us that it is not part of their science.

§ 3. *The fundamental constants satisfy the conditions of stability of the system.*

Since relations exist between the values of the fundamental constants, we may ask for the most general expression of these relations. The principal of these relations are, by Lagrange and Laplace, proved to be such as to fulfill the conditions of stability of the system. These conditions are—

1. *Incommensurability of the times of rotation*, ensured by the distances forming an exponential series (1).

2. The *central mass* vastly preponderating, and the greatest masses revolving where the mutual distances are the most considerable.

3. The *direction of all motions* is the same.

4. *The plane of all orbits is and remains nearly the same*, because

$$\Sigma m \sqrt{a} . tg^2 i = c_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is and remains a small quantity (the letters having the same signification as in § 1).

science. Less arrogant, because more true, he is when writing to Burnett (about 1680–81), "but yet I must confess I know no sufficient cause of y^e earth's diurnal motion."

Even Biot, notwithstanding his blind admiration, above cited, cannot help seeing something *more than* gravitation can account for, when noticing the harmony in the rotary and translatory motions. He says: "cet accord, *qui tient sans doute au premières causes qui ont déterminé les mouvements planétaires, est un des phénomènes les plus remarquables du système du monde.*—Astron. physique, vol. iv, chap. v, p. 467.

5. *The eccentricity, e, of the orbit is and remains nearly the same,* because

$$\Sigma m \sqrt{a} . e^2 = c_2, \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

is and remains a small quantity.

6. The *density, d,* is such that the diameter of the bodies is small in comparison to their distances.

We may add—

7. The *form* of the planets is such that the influence of the deviation from a sphere is the smallest possible.

§ 4. *Gravitation is insufficient.*

The laws of Kepler are grand—as well as Newton's theory in accounting for them; but the above laws of Lagrange and Laplace are certainly of a superior order, and the theory of gravitation in failing to give even a shadow of a reason for these laws proves itself to be not the whole truth: we must go beyond this force!

Astronomers seem to forget the *history* of their own science; for how could they otherwise deny the legitimacy of accounting for the fact that the above laws express the stability of the world? Had not astronomers at the time of Kepler the same reason to be satisfied with his laws as astronomers have now in abiding by the laws of stability? And is it not as urgent to discover the causal connection between these laws ensuring great *duration* to our system, as it was to find in gravitation the mechanical cause of those laws ruling the spheres at the time being?

Now the hypothesis of Kant and Laplace will be found to account for the laws of stability as rigidly as the hypothesis of Newton accounts for the laws of Kepler; why, then, deride the former and adore the latter hypothesis? Or do we even forget that "the principle of gravitation" is but a "*hypothesis?*" Did not Newton himself consider it as such? Is not this force fully as mysterious and fully as much beyond the reach of direct observation as the chaos of Kant and Laplace? The former we *assume* as continually acting, because we find that *the motions* are such as this force would produce (*provided* a tangential force, of which gravitation knows nothing, also acts in a definite manner, etc., etc.). Why not also assume the latter as having been real, if we find by the same mechanical deductions that the existing harmony, as expressed in the stability of the system and ensured in the mutual relations of the fundamental constants follows directly from the above-named chaos? If in the one case we reason from fact or law to cause—why not also in the other, provided our conclusion is as legitimate?

Here astronomers will not fail to object that this last condition

is *not* satisfied. We fully admit this; but beg them to remember that it has taken two centuries of labor to ensure this legitimacy to gravitation—that Newton did not leave gravitation as a mere suggestion (as such it had existence before him), but in his immortal *Principia* gave the necessary mechanical firmness to this hypothesis: how different has been the lot of Kant and Laplace's hypothesis! The first of these expounded it rather fancifully in connection with speculations on the inhabitants of distant globes;[8] the latter only gave a few bold and deep outlines of the nucleus of the theory! What would to-day be the estimation of gravitation if we, instead of Newton's *Principia*, only had a few of Hooke's sublime guesses, if these had only been considered by men like Fontenelle[9] instead of being investigated by Euler, the Bernoullis, Lagrange, Laplace, Gauss, Hansen, Plana, etc.?

Can anything be more unjust than exaltation of the hypothesis of Newton—this deservedly cherished subject of the master minds of two centuries—above the hypothesis of Kant–Laplace, which, being too early left even by its astronomical parent, has been ever since considered an outcast in the world, endangering the reputation of any one who would dare to touch it?

We will adopt this almost forlorn hypothesis as a *mere hypothesis*—we will patiently and carefully trace its bearings by means of as rigid an analysis as we can command in this most intricate field; we will minutely compare the results thus obtained with the actually observed state of things; and if we find *the correspondence between idea and phenomenon*, between analysis and observation, to be very close, we hope that those who have analysis more at their command than we, will pay as much attention to this high branch of astronomy as has been, and deservedly continues to be, bestowed on Newton's hypothesis of gravitation. If our feeble endeavors only succeed in making Kant–Laplace's *hypothesis* admitted as such among analysts, we shall have accomplished all we desire: for then this hypothesis will soon be considered as firmly established a principle as gravitation,[10] or as the fact of the rotation of the earth,[11] which latter—notwithstanding Foucault's pendulum and gyroscopes—still remains unproved by *ocular evidence*, all "demonstrations" being in fact

[8] Allgemeine Naturgeschichte und Theorie des Himmels, 1755.

[9] Théorie des Tourbillons Cartésiens avec des réflections sur l'attraction (1752). Also his Entretiens sur la pluralité des mondes.

[10] Gravitation was always treated as a mere hypothesis by the Cartesians; the work of Fontenelle above cited offers the instance most generally known.

[11] The scientific prejudice existing against the nebular theory is perhaps as injurious as the religious prejudice once "*resisting*" the motion of the earth. See the amusing statement in the preface to the 'Dei massimi sistemi,' (ed. Padova, 1744,) that the "'Moto della Terra' non può né dee amettersi se non come pura ipotesi matematica, che serve a spiegare più agevolmente certi fenomeni." Also Galileo himself, in his Dialogue (*Opere compl.* Firenze. Vol. i, 1848, pp. 387 and 447).

inductive, reasonings with our senses. Thus, in the latter, we *see* the plane of the oscillations rotate, but *conclude* it to be the earth. It requires at least as much mental effort to apprehend its true bearing as the simple reference to the diurnal motion of sun, moon, and stars.

§ 5. *How far the nebular theory accounts for the stability of the system.*

The theory of gravitation can never, therefore, account for the stability of the solar system. How far, then, does the nebular theory explain those great fundamental conditions of the system that ensure not only the harmony of the solar world but even make this harmony (almost) permanent?

In order to invite physicists and astronomers to the perusal of the following introduction, we will try to give a simple answer to this most important question.

I. *The plane of all planetary orbits must be nearly the same* (see § 1, 9, and § 3, 4, also § 10).

This theorem has been clearly seen by Kant and Laplace; it is the most immediate expression of the hypothesis. Mr. Trowbridge has recently pointed out[12] to us a very interesting consequence hereof, viz: the most distant planet *must move in the invariable plane;* and, indeed, the inclination of the orbit of Neptune is 1° 47′, that of the invariable plane 1° 41′.

II. *The direction of all planetary revolutions must be the same;* this obvious consequence of the hypothesis accounts for § 3, 3, and § 1, 10.

III. *The eccentricity of the planetary orbits must be very small*—accounting for observation, § 1, 11, and condition of stability, § 3, 5 and § 10.

This proposition has been deduced in general reasonings by Kant and Laplace; in the following we will try to give a demonstration of it.

IV. *The planetary distances are such that the successive planets were evolved at equal intervals of time;* or if t=1, 2, 3, ... respectively for Mercury, Venus, Earth ...., then the distance is

$$a_t = \alpha + \beta . \gamma^t, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where α, β, γ are constants, and t *the age of the planets* above that of Mercury. From this follows the condition of stability that *the periodic times are incommensurable* (§ 3, 1). Besides, it is seen that this law accounts for the empirical law of Bode (§ 1, 8).

The analytical demonstration of this law is one of the principal objects attempted in the present introduction. (See § 13.)

V. *The mass of the more distant planets is the greater*, on account of the greater space from which the material of the planet was condensed (space increasing according to IV). This is confirmed

[12] On the Nebular Hypothesis, § 24; this Journal, November, 1864, page 355.

by the fact that the sum of the *four great exterior planets* is 480 times the mass of the earth, while the sum of the masses of the *four interior planets* is but twice the mass of the earth. Furthermore, *the mass must increase toward the sun*—as the density from which the rings were formed was greater nearer the center. This is confirmed, as the mass of Neptune is 25·6, the mass of Uranus but 14·5, so that the mean of the two most distant is 20; the mass of Saturn is *five* times as great (101·6), and the mass of Jupiter, again, three times greater (339·2). The minimum in the case of the mass of Uranus is evidently produced by the simultaneous influence of both the above principles.

The mass of Jupiter is a maximum; it is so great that *the next following ring was broken up into fragments by the perturbing influence of so stupendous a mass*—thus originating the host of *asteroids*, and perhaps also the *meteorites*.

The very small mass of the interior planets as compared with the exterior ones is not astonishing, if we remember that the inter-planetary space between Jupiter and Saturn is to that between Venus and the Earth, as $10^3 - 5{\cdot}2^3$ to $1{\cdot}00^3 - {\cdot}72^3$, or as 860 to ·63, or nearly as 1300 to 1. The mass of Jupiter is to the mass of our earth as 340 : 1, thus giving us still some margin for an increase in density toward the center of the nebula.

Being as yet unable to give the precise *theoretical law of the masses*, we are obliged to make the above few suggestions, in order to show that the nebular hypothesis at least gives a general law of distribution of the planetary masses in conformity with § 1, 1, and § 3, 2. (Prof. Kirkwood takes a similar view of the asteroids; see this Journal, 1852, xiv, 214.)

VI. *The figure of the planets* (being a *condensed* vapor) *must be an oblate spheroid* of

VII. *Small ellipticity*, because

VIII. *The velocity of rotation is but small* (compare § 1, 2, 3, 7, and § 3, 7).

This last proposition is based upon the fact that the moment of rotation is but the *difference* between the moment of revolution of the exterior and interior part of the planetary ring.

Still, *the exact amount of this velocity*, as well as the period of rotation of the sun, has not yet been deduced from the nebular hypothesis; we have often attempted it, but as yet have not been able to solve this difficult problem.

Prof. Kirkwood has found[13] the empirical law of the velocity of rotation, *a law analogous to the third law of Kepler*. We have repeatedly arrived at expressions similar to (but not identical with) Kirkwood's law.

IX. *The Plane;* and

[13] As above, this Journal, 1850, ix, 395.

X. *The direction of rotation* of the planets has been considered at variance with the nebular theory, ever since the discovery of the lunar system of Uranus. We believe that our analysis of this problem[14] shows that the rotation of Uranus and Neptune, both as to position of the axis and direction of motion affords a very interesting confirmation of the theory. See § 1, 5, 6.

XI. *The density* of the planets has also been considered as being adverse to the theory; but if, as necessary, the influence of the age is taken into account, it is found that the minimum density exhibited by Saturn is demanded by the theory.[15] Compare § 1, 4, and § 3, 6.

XII. *The number of satellites* was already shown, by Kant, to increase with the distance from the center of the nebula. Though not usually given as such, it nevertheless is a condition of stability of the system—at any rate it is conformable to observation (§ 1, 12).

The rings of Saturn are best considered as a host of satelloids, corresponding to the planetoids (and meteorites) of the solar world—thereby accounting for the excessive thinness and the subdivisions of the rings.[16]

In looking back upon the preceding account of the present aspect of the nebular theory, it will be seen

A. That the four great fundamental conditions of stability referring to *the system at large* are now satisfactorily deduced from the hypothesis of Kant–Laplace (I–IV above).

B. That the problem of the *mass* (V) and the *number of satellites* (XII), though not completely evolved, still is sufficiently comprehended to enable us to say that the analytical solution is *possible;* and

C. That the elements referring to the *single planets*, or rather their subordinate systems, are, with the exception of the *exact* law of rotation (VIII), fully deducible from the fundamental hypothesis of Kant and Laplace.

We see, then, that the *fundamental constants* of the solar system, which number about seven hundred (§ 1), exhibit very remarkable mutual dependencies (§ 2), which are such as ensure the permanence or stability of the system (§ 3), which Newton's law of gravitation cannot account for (§ 4). Though they offer a higher problem for theory than Kepler's laws, astronomers have hitherto been unwilling to recognize the analysis of the above conditions of stability as part of their science. Laplace, while instrumental in bringing to light the great laws of the stability of the system, independently reproduced the bold hy-

[14] On the density, rotation, and relative age of the planets; this Journal, 1864, xxxvii, 36, 48.

[15] As above, p. 49.

[16] On the density, etc.; this Journal, 1864, xxxvii, 54.

pothesis of Kant, and though this has been most grievously neglected by analysts and astronomers, still it now affords us a full solution of the four great harmonies ensuring the permanency of the solar world, and also solves most, and at least indicates the solution of, all other problems relating to the harmony of the fundamental constants of the solar system.

May we not hope that astronomers will begin to bestow on this theory some share of their labor?

§ 6. *The Hypothesis.*

We assume, with Kant and Laplace, as the primitive condition of the solar system, or as *nebula:*

The space of the solar system was filled with matter having a moment of rotation.

This *matter* is endowed with the same forces we know it to possess; a simple calculation will furthermore show that it was a highly rare vapor. Its chemical constitution we will leave out of consideration for the present; we therefore consider it as composed of the elements we know here on earth, many of which we *now* know to be found on the sun, and are probably also on the distant stars;[17] still there can be no doubt but that many more elements exist than we are acquainted with. Many of the spectral lines even of our own central star are irreducible to spectra of known elements. We therefore mean simply to say that at the above primitive period the *elements had been created.* I hope at some future time to publish an attempt at a *mechanical theory of the elementary bodies*, which has occupied my time for about ten years, and wherein I endeavor to show the physical properties of the known elements to be *definite functions* of their *atomic number and form.* Accordingly, there would yet remain a more primitive condition, the existence of the *one primitive matter* (Urstoff) which would be considered as the direct creation of the *one* God.

The *rotation* of the nebula is *not* to be thought regular, but simply amounting to a certain momentum. I have elsewhere[18] tried to show that such rotation may be considered as the *effect of a difference of any kind between the primitive forces of attraction and repulsion* wherewith we know matter to have been endowed.

If, therefore, these views should be well founded, we should have arrived at the grandest principle we can conceive of in the present state of our knowledge; we should be able to see how from *created matter alone the whole of the solar system has been developed;* we would be enabled to conceive the almighty *fiat* as

[17] Rutherford, Astronomical Observations with the Spéctroscope; this Journal, 1863, xxxv, 71. Above all, Bunsen's and Kirchhoff's memoirs on their great discovery.

[18] This Journal, 1864, xxxvii, 52.

one single act. How much such a theory would tend to elevate our conceptions of the great Author, we cannot here develop.

In the present paper I shall not go farther back in time than to the existence of the nebula of Kant and Laplace as above defined.[19]

§ 7. *Plateau's experiment.*

Before entering upon the analysis of the nebula, we must refer to the experimental evidence of the nebular theory afforded by the beautiful experiments of Plateau, detailed in his *Mémoire sur les phénomènes que présente une masse liquide libre et soustraite à l'action de la pésanteur*, Pt. I (Neuv. mém. de l'Acad. de Bruxelles, vol. xvi, 1843). His results are:

1. A liquid, subject only to the action of its molecular forces assumes the form of a perfect sphere (§ 2).

2. This globe is flattened at its poles, if subject to rotation,

[19] It is perhaps not out of place here to give a synopsis of the different distinct *ages* that are characterized by a further individualization or a new direct creation according to the views above indicated.

Three (or four) direct acts of the Deity may be recognized, viz: the creation of *matter*, of *life*, of *mind* (and the *redemption*). The formation of the *elements* out of matter characterizes the *first age;* the formation of the *solar world*, with its planets, moons and central sun, the *second age;* while the *third age* beheld the development of our *earth* from a vaporous ball to its present shape; in the *fourth age, life* was created in the form of *plants* and *animals;* in the *fifth age*, mind, the investigating mind, was introduced by the creation of *man*, the *cephalized animal;* while a *sixth age* will behold the destruction of the whole system, occasioned by the *extinction* of the solar body and the *resistance* of ether. (See this Journal, 1864, xxxvii, 56.)

To *every age* correspond two sciences: the first relates to the development—Whewell would call it the *Paletiology* of the age—while the second relates to the actually *existing product* of the development or creation of that age, i. e., *the* science. Thus we obtain the following general view of the natural sciences:

Age	1	2	3	4	5	(6)
	I. *Creation of* MATTER. Development of			II. Creation of	III. Creation of	Destruction of the world.
	Elements.	*Solar system.*	*Earth.*	*Life.*	*Mind.*	
Paletiology	Atomology	Planetology.	Geology	Paleontology.	Archeology.	
Science.	Physics, Chemistry.	Astronomy.	Geography.	Botany, Zoology	History.	

These names have of course to be taken in their widest sense; thus geography stands for physical geography, meteorology, etc., and history comprehends not only political but also the intellectual history of the human race, thus including again *all* the sciences in their historical development. We see how "planetology" is allied to geology and astronomy.

3. If the rotation becomes sufficiently rapid, the *whole globe* is transformed into a *ring* in the equatorial plane (§§ 11–14) which, by continuing the rapid rotation of the large central disc, even loses its regularity, and separates into small masses which immediately assume globular forms. "But," continues he (§ 19), "this is not all; one or more of these spheres are always seen to take, at the instant of formation, a motion of rotation on their own axes, and this motion is almost always in the same direction as that of the ring."[20] He even found that still smaller globes were formed.

4. If a small disc is put into very rapid rotation, a ring is formed, while a part of the original globe remains on the axis (§ 21), so that Plateau rightly concludes with saying (§ 27) that most of the phenomena of the relative configuration of the heavenly bodies have been reproduced by him on a small scale.[21]

§ 8. *The condition of the primitive Nebula.*

Whatever may be the distribution of matter in the nebula, and however the particles may move, the beautiful theorem—the magna charta of the nebular theory—obtains, i. e. *the nebula possesses the invariable plane of maximum areas*, or if ω represents the angular velocity, r the projection on the invariable plane of the radius vector drawn out from the center of gravity, m the mass, and A the projection of the area swept over by r in the unit of time, we have, C being a constant,

$$\Sigma m\mathrm{A} = \mathrm{C}\,; \quad . \quad . \quad . \quad . \qquad (5)$$

or, as the *centrifugal force* γ of the particle m with respect to the principal axis is

$$\gamma = \omega^2 r \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

while, at least for a very small unit of time,

$$\mathrm{A} = \tfrac{1}{2}\omega\, r^2, \quad . \quad . \quad . \quad . \qquad (7)$$

we have also

$$\Sigma\, m\, r^{\frac{3}{2}}\, \gamma^{\frac{1}{2}} = 2\mathrm{C}. \quad . \quad . \quad . \qquad (8)$$

On account of the resistance of the ether, C will not be quite constant, but decrease in time; still it is apparent that, as a first approximation, we may neglect this resistance by considering C strictly constant.

Now, by the mutual attraction of the particles, the nebula is continually becoming more dense, or r is continually decreasing; hence, by (8), *the centrifugal force of any particle in the nebula is continually increasing.*

[20] Of course; for in this oil the density is the same. See our article, this Journal, 1864, xxxvii. 51, $\delta = 0$.

[21] Malgré la difference des lois que suivent les forces attractives dans ce cas et celui des grandes masses planétaires, nous avons vu se produire, en petit, une représentation frappante de la pluspart des phénomènes de configuration relatifs aux corps célestes.

But the force of gravity at the surface is likewise constantly increasing; for we may without materially erring conceive the mass below the particle to remain constant, but then *gravity* is inversely as the square of the radius, or *rapidly increasing with the progress of condensation.*

But *these two forces determine the figure of the Nebula.* However irregular the figure may be at first, we see that the moulding forces, by constantly increasing, will at length shape the nebula accordingly. From Plateau's Experiments (see above, § 6, result 1, 2) we know this shape to be a flat ellipsoid. Laplace[1] has demonstrated that but *one single oblate ellipsoid of revolution* will be produced by these forces, *i. e.*

$$z^2 + m\,(x^2 + y^2) = a_1^{\,2}\,, \quad . \quad . \quad . \qquad (9)$$

the plane x, y, coinciding with the *invariable plane*, being the equator, z the *axis of rotation* $= 2a_1$ and

$$m = \frac{1}{\sqrt{1+\lambda^2}}, \qquad \lambda = tg\,\theta, \qquad \sin\theta = e, \quad . \qquad (10)$$

e being the *eccentricity* of the meridian; hence the equatorial semi-axis

$$a = a_1\sqrt{1+\lambda^2}. \quad . \quad . \quad . \qquad (11)$$

Assuming, for a moment, the nebula to be homogeneous, we can determine the eccentricity by the *density* δ, and the *angular velocity* ω (i. e. by (6) proportional to the square root of the centrifugal force g at a unit of distance). Laplace found, if the mass of the whole nebula be M, and its moment of inertia E, that

$$E = \tfrac{4}{15}\pi\delta a_1^{\,5}(1+\lambda^2)\sqrt{g} = \frac{\sqrt{g}}{5}\,a^2 M\,; \quad . \quad . \qquad (12)$$

$$M = \tfrac{4}{3}\pi\delta a_1^{\,3}(1+\lambda^2) = \tfrac{4}{3}\pi\delta\,\frac{a^3}{\sqrt{1+\lambda^2}}\,; \quad . \quad . \qquad (13)$$

$$q' = \frac{25\,E^2}{M^{\frac{10}{3}}}\left(\tfrac{4}{3}\pi\delta\right)^{\frac{1}{3}}; \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

$$\text{arc}\,(tg = \lambda) = \frac{9\lambda + 2q'\,\lambda^3\,(1+\lambda^2)^{-\frac{2}{3}}}{9 + 3\lambda^2}; \quad . \quad . \qquad (15)$$

which last equation he shows to have *but one* single positive real root, so that λ or e has but one value, if g, the centrifugal force, and δ, the density, are given.

But the density is certainly *not* constant throughout the whole nebula; but as this nebula is a gaseous body, δ will be determined by γ and gravity and, just as in the case of our earth, be-

[1] Mécanique Céleste, Liv. iii, chap. III, § 21.

come uniform in the successive homothetic ellipsoidal shells included between any two successive surfaces. Hence we may consider δ as a function of the equatorial axis of such surface, and, as the density is increasing toward the center, we may, instead of the general law

$$\delta = f(a), \quad . \quad . \quad . \quad . \qquad (16)$$

take the law assumed by Laplace for the interior of our earth,

$$\delta = \Delta - c\,a. \quad . \quad . \quad . \quad . \qquad (17)$$

Plana has shown[2] that this law is the most probable in the case of the earth. Prof. Forchhammer of Copenhagen has lately shown[3] how this law accounts for one of the principal circumstances relating to the succession of geological strata.

It must finally be borne in mind that the nebula may have the various elements at the same place, because the laws of diffusion of gases will apply to the gaseous nebula. Thus far the chemical analyses of meteorites and the spectral analysis of the sun, moon and planets have corroborated this conclusion; still we must not hastily conclude that *some* differences may not obtain,

[2] Note sur la densité moyenne de l'écorce superficielle de la terre. *Astronomische Nachrichten*, 1851, vol. xxv, No. 828. For the density at the center of the earth, he finds 16·3010.

[3] Inledning til en Række af Forelæsninger Stoffernes Kredslöb i Naturen (on the circulation of the elements in nature.) *Nordisk Universitets-Tidskrift,* viii, 1 hefte. Copenhagen, 1862. p. 68–81.

As an instance, *Forchhammer* describes the circulation of *Lime:* first the smaller diurnal orbits between plants and animals and the soil; next the greater annual circulation between land (washed by rain) and the sea; and, being here deposited in large strata through the agency of marine animals, these are again, after long geological periods, put into circulation, especially by the inorganic powers of nature, for these again to recommence a new cycle.—Now, where did the lime at first come from? Forchhammer thinks that as granitic rocks (specific gravity 2·7) are less dense than the dark trap-rocks, (on Bornholm, sp. gr. up to 2·93,) they would, during the first period of the igneous earth, float upon the trap; thus the first solid shell would be formed of granitic rocks, i. e. *free from lime*—consequently unfit to support life, not fossiliferous. By the next revolutions this rock was dislocated and broken through by the underlying trap-rocks containing lime and iron as silicates; the atmosphere being so rich in carbonic acid and being dissolved in the then hot waters would decompose these silicates, and thus bring lime into circulation. Organic life can now first commence—and the first fossiliferous rocks' appear. This beautiful idea is further substantiated by the fact that volcanos, after a longer period of rest, commence their eruption with emitting trachytic, i. e. granite-like, masses almost free from lime—which are later succeeded by the *heavy* black lava, containing both lime and iron.

The richest deposit of gypsum, the Triassic period, is succeeded by the extraordinary limestone formation of the Jurassic period, thus giving another link in this chain of inductions, for gypsum, being more soluble, will more rapidly circulate, and thus occasion a greater deposit of limestone.

These views of our illustrious teacher show us what patient investigation yet may accomplish; we see the cause for the succession of granite and trap—see why organic life could not commence earlier than it does—see the cause for abundance of limestone during the Jurassic period, etc.—in the simple circumstance that granitic masses, being lighter than trap, were exterior to the latter in the igneous globe.

as the diffusion certainly is limited by the sinking of the denser particles. In a nebula from which a whole *cluster* of solar systems has been formed, we may therefore expect to find considerably different elements. We thus decline the imputation of Rutherfurd that homogeneity of original diffuse matter "is almost a logical necessity of the nebular hypothesis," and cannot see any real objection to this hypothesis, if, as he says, "we have now the strongest evidence that they (the stars) also differ in constituent materials" (this Journal, 1863, vol. xxxv, p. 77).

In regard to the signification of δ we must remark that, in the following, we use the letter δ to represent *the mean density of the nebula* from the centre to the distance r, while in (17) δ indicates the density of the shell at the very distance r. As (17) is only adduced to serve for a comparison, this course is legitimate. But it is easily demonstrated, that, at least for a spherical nebula, this law (17), if true for the individual shell, will also be true for the mean density of all shells inside of it. For, the actual density varying according to (17), the mean density of the interior body from $r=0$ to r is found to be

$$d = \varDelta - c'.r, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where $c'=\frac{3}{4}c$. This law[4] is evidently the same as (17).

§ 9. *Attraction in the Nebula.*

As the nebula now may be considered made up of homothetic oblate ellipsoidal shells, individually of constant density, and as we know (from *Méc. Cél.*, liv. iii, chap. I, § 2,) that such a shell does not exert any attraction on a point within, we find *the attraction at any point in the nebula determined by the attraction of the ellipsoid whose surface passes through that point.*

This force, at the point x, y, z, is given by the following formulæ (from *Méc. Cél.*, liv. iii, ch. I, § 4), independent of the law of the density (17), and merely depending on the proved uniformity of the density in each separate shell.

If we put

$$Q=\frac{3m}{2\lambda^3}\left[\tan^{-1}\lambda-\frac{\lambda}{1+\lambda^2}\right], \quad . \quad . \quad . \quad . \quad . \quad (19)$$

then the components X, Y, Z, of the attraction (positive toward the origin) are

$$X=Q.\frac{x}{a_1^3}; \qquad Y=Q.\frac{y}{a_1^3}; \quad . \quad . \quad . \quad (20)$$

$$Z=\left(Q-\frac{\lambda^3}{1+\lambda^2}\right).\frac{z}{a_1^3}. \quad . \quad . \quad . \quad . \quad . \quad (21)$$

[4] The "Density" in Trowbridge's article (this Journal, xxxviii, 354, 1864), is different, because referring to the density at different periods of time.

All of these three components act to *condense* the nebula; but X and Y also determine the revolution of the particles, while Z has no such influence, all motions in the direction of the axis of z mutually destroying each other, because x, y is the invariable plane. Composing X and Y we get

$$R = Q . \frac{r}{a_1^3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and *directed toward the axis of rotation*; $r^2 = x^2 + y^2$.

Substituting the first (13) ($m = M$) in (22) we obtain

$$\left. \begin{aligned} & R = \mu . \delta r, \\ \text{where} & \\ & \mu = \frac{2\pi}{\lambda^3} [(1 + \lambda^2) \text{ arc } (tg = \lambda) - \lambda]. \end{aligned} \right\} \quad . \quad (23)$$

As now μ only depends on λ, i. e. on the eccentricity (10) which is constant, the shells being homothetic, we see that μ is at any given moment for all parts of the nebula the same, hence: *the radial force R in the nebula is proportional to the density δ and the distance r from the axis of rotation.*

This simple result is of very great importance, as we shall see in the sequel.

§ 10. *The orbit of the Planets.*

The particles of the nebula had originally motions in all directions; but as we assumed the existence of a momentum of rotation (§ 6), the principle of the invariable plane will keep up this momentum (§ 8), while all motions at variance therewith will in time mutually destroy themselves. Therefore, *all particles describe circles around the axis of rotation.*

Such as the orbit of the single particles that formed a planet will also be the orbit of the latter; hence the eccentricity and inclination of all planetary orbits ought to be zero. This may also be seen from Plateau's Experiment, and agrees well with the smallness of both the eccentricity and the inclination (see §1, 9° and 11°, § 3, 4° and 5°). Still, neither of these two quantities is actually zero. Are, then, these small deviations from this value accounted for by some accessory conditions of the problem?

We think so, for there are *two* modifying circumstances, the *rupture of the ring*—which it is beyond our power as yet to take into consideration—and the perturbating influence of already separated masses. The latter we may estimate. Representing the eccentricity by e, the inclination of the orbit to the ecliptic by i, to the invariable plane by I, we have from observation [*Humboldt's Cosmos*] :*

* The numbers in the last column of the following table are not quite exact.—Eds. Jour. Sci.

	e.	i.	I.
Mercury,	·2056	7° 0′	5° 19′
Venus,	·0068	3° 23′	1° 42′
Earth,	·0168	0° 0′	1° 41′
Mars,	·0932	1° 51′	10′
Asteroids,[5]	·160	7° 55′	6° 14′
Jupiter,	·0482	1° 19′	13′
Saturn,	·0561	2° 30′	48′
Uranus,	·0466	0° 46′	55′
Neptune,	·0087	1° 47′	6′
Invar. plane,	....	1° 41′	0° 0′

We see how clearly the principal members of the system *move in one plane*, and that this plane is the *invariable plane* of the system; the great planets deviate less than one degree, the principal of the interior planets, Earth and Venus, only $1\frac{2}{3}$ degrees—and even the inclination of the smallest planet, Mercury, amounts to but $5\frac{1}{3}$ degrees! So also in relation to the eccentricity, this being less than one-twentieth for the principal bodies.

As to the deviations, we see that Neptune, which if not the most distant planet, certainly is (or *was*) separated from the next by a very large distance, so that if either could not at all, or but slightly, be disturbed, has indeed the smallest inclination (only 6 minutes!) and about the smallest eccentricity (less than one-hundredth!). Jupiter, which, on account of its enormous mass, could not be much disturbed by other bodies, has an inclination of only 13 minutes, while Saturn and Uranus have—corresponding to their smaller mass—about four times as considerable an inclination (48 and 55 minutes). The eccentricities of these three orbits are about equal; perhaps that of Jupiter is near its maximum, or the eccentricity of Saturn and Uranus near their minimum.

The inclination of the Earth and Venus is greater than that of the exterior planets, for the mass of the former is small as compared to that of the latter; but as Venus and the Earth are the great planets among the interior, we see that the inclination and eccentricity of Mercury's orbit are much more considerable than either, and that Mars has less inclination and eccentricity than Mercury. Is it because Jupiter, the only planet that would exert considerable perturbation on its development, was so far distant?

The orbit of the asteroids is explained in § 5, V.[6] We gave publicity to these views in an address delivered before the physical section, at the meeting of the Scandinavian philosophers, July, 1860.

[5] Mean of *the first* 72 *Asteroids*, elements given in Table of Smithsonian Report, 1861, p. 218–219.

[6] We intended in this place to give a fuller account of our views concerning the development of the asteroids; but learning from a letter of Mr. Trowbridge that the continuation of his article will contain a solution of this problem, I abstain for the present from publishing my details.

The same principles will apply to the satellites; but we have too few data to make a comparison of this principle with observation profitable.

§ 11. *The periodic time of the Planets; Kepler's third law.*

Since every particle in the same shell revolves around the axis under the influence of a force R proportional to the distance r from the axis (§ 9), we know from mechanics that the periodic time T of such a particle is

$$T=\frac{2\pi}{\sqrt{\mu\delta}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where, it will be remembered (23), μ is the same for the whole nebula, and δ constant for the same shell, so that *the time of revolution is the same for all particles of the same homothetic shell, but for the different shells inversely proportional to the square root of the density.* Thus every shell rotates as if it were solid; and if the whole nebula had the same density throughout it would rotate like one solid. But if the density be different in different parts, some shells will rotate faster than others (§ 12).

Eliminating δ by means of (13) we get

$$\left.\begin{aligned} T^2 &= \mu' \cdot \frac{a^3}{M} \\ \mu' &= \frac{8}{3}\,\frac{\lambda^3}{\sqrt{1+\lambda^2}}\,\frac{1}{(1+\lambda^2)\tan^{-1}\lambda-\lambda} \end{aligned}\right\} \quad . \quad (25)$$

We know that the ellipticity of the nebula is determined by the centrifugal force, and the latter by the state of condensation (§ 8); and even in case an ellipsoid becomes impossible, we can not but conclude that the figure continues to be determined in the same manner. But the condensation continues—the increase of the centrifugal force depending thereon will also continue and produce a series of rings in a certain succession, just as *one* ring was formed in the experiments of Plateau (§ 7). We see now how the continued increase of the condensation occasions a periodical change in the figure of the nebula. Granting the variation of the figure beyond the possible ellipsoid to be determined by the same circumstances as the ellipsoid itself, we may compare the corresponding stages of the nebula by referring to the same ellipticity e or the same μ' in (25); at any rate, we know that this can be done if we only compare the nebulæ, when within the limits of the possible ellipsoid. But then μ' will be the same for all rings, and as the mass of the planets is but very small as compared to that of the sun, M remains *almost* constant. Then (25) becomes

$$\frac{T^2}{a^3}=\text{constant}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

or *the squares of the times of rotation of the different rings are as the cubes of their radii.*

If we remember that the possible ellipsoids reach to a proportion of 1 to about 3 between polar and equatorial diameters of the nebula, we can be sure that *this* covers the principal part of the metamorphosis; hence, (26) is rigorously proved for the greatest part of the condensation intervening between the formation of two successive rings; the nebula acquires its principal dimensions while changing in accordance with the ellipsoidic figure, and when abandoning this it quickly passes to the form of a slightly oblate spheroid and a ring. The interruption in our strictly mathematical demonstration cannot, therefore, seriously interfere with (26). But then this or *Kepler's third law is a consequence of the nebular hypothesis*, or *the observations embodied in this law sustain equally the nebular hypothesis and gravitation.*

Again, inductively, we may conclude from Kepler's third law that the interruption in our analytical deductions occasioned by our ignorance of the exact mechanical laws of the metamorphosis of the ellipsoid into the *globe ring* (we might in reference to Saturn find the expression *Kronion-form* convenient) is not of serious consequences.

Thus we may at least conclude from the third of Kepler's great laws that the development of the planets was periodical; for, this law being a fact, and (25) being rigorously true, we *must* have

$$\frac{\mu'}{M} = \text{constant}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

but, as remarked before, M remains essentially constant, hence μ or what is the same λ, i. e. the ellipticity ϵ of the nebula corresponding to the different planets, must have been the same at corresponding epochs, just as we assumed above.

But *if* the metamorphosis of the nebula has been periodic, and not simultaneous, we must ascertain *whether the successive intervals of time were equal or not.* We shall find that *they were equal*, just as it would be the most natural or the simplest to assume.

§ 12. *Spiral Nebulæ.*

In the preceding paragraph we considered the density of the nebula sensibly equal throughout, so that the nebula always rotated like a solid, all particles having sensibly the same period of revolution. This might be done, because the dimensions of such nebula—however immense in reality—are not sufficiently great to produce a very large change in δ (17) in the space allotted to each planet.

But there may be bodies of dimensions so vast as to render it utterly impossible to consider the density approximatively uni-

form throughout the nebulous mass. Then the nebula will *not* rotate like a solid, but the angular velocity ω of any particle will be

$$\omega = \frac{2\pi}{T}, \quad \ldots \ldots \ldots \ldots (28)$$

or, by (24),

$$\omega = \sqrt{\mu\delta}. \quad \ldots \ldots \ldots \ldots (29)$$

As μ (23) is constant for the whole nebula, we see that *the angular velocity is proportional to the square root of the density*, or, according to (17), greatest near the center of the nebula.

If θ be the angle of position of those particles which are originally (i. e. when $t = 0$) in one and the same straight line, we have at the time t,

$$\theta = \omega t, \quad \ldots \ldots \ldots \ldots (30)$$

or by (29)

$$\theta^2 = \mu . \delta . t^2, \quad \ldots \ldots \ldots \ldots (31)$$

Remembering that the density is a function of the distance (16) and also of the time on account of the progressing condensation, we see that (31) may be written,

$$\theta^2 = \mu t^2 f(a, t) \quad \ldots \ldots \ldots \ldots (32)$$

At any given moment of time (t constant), all the particles that originally were situated in the same straight line given by

$$\theta = 0 \quad \ldots \ldots \ldots \ldots (33)$$

will now form the curve

$$\theta^2 = \varphi(a),$$

i. e. *a spiral.* This contains the fundamental principles of a mechanical theory of the *spiral nebulæ.*

Substituting Laplace's law of the density (17) in (31) or (32), we obtain as the equation of the spiral

$$a = \frac{\alpha^2 - \theta^2}{C}, \quad \ldots \ldots \ldots \ldots (34)$$

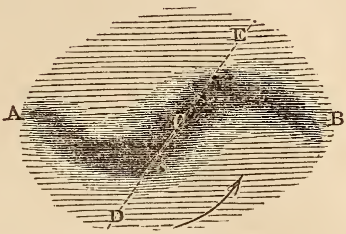
wherein $\alpha = \mu . \Delta . t^2$ depends upon the ellipticity (μ), the *density* Δ *at the center*, and the time t, whilst $C = c\mu t^2$ depends upon the same μ and t and the *rate of variation of the density.* We see that these spires are limited, for $0 < \theta < \alpha$; and that the sweep α of the spire increases with the *age* of the nebula, the *density* at its center, and the *ellipticity.*

In order that such spiral structure may become apparent in a regular ellipsoidal nebula, the brightness must originally have been different in different meridians, though the density was constant in the same shell, i. e. the same in all meridians. Thus, if the brightness in the nebula was originally greatest in the opposite meridians AC and BC, (Fig. 1.) and it rotates in the direction of the arrow around the axis C, the *spiral nebula,* (fig. 2,) would result. As the age increases, the sweep, or the angle

BCE $= \alpha$, would increase, whilst A and B remain nearly at the same distance from C: so that an annular nebula with a central core might in time result from a spiral nebula; even several concentric rings might be formed.

1.

2.

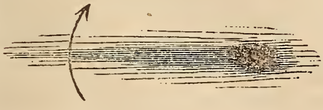


We cannot suppose any nebula to have different brightness in parts of the same density; and neither is it reasonable to assume such vast masses to be already shaped to a regular ellipsoid by the influence of the central forces (see 7).

It is much more reasonable to think that the nebulous masses at first were of *any shape*—such as might result from a predominating attraction of those portions where the heaviest elements were formed or collected in greater abundance. Then the formulæ deduced in the preceding paragraphs, though no longer representing the exact conditions of the nebula, still would continue to be approximate; the angular velocity would still be greatest near the central parts, as can also easily be shown directly, by considering the motion of each particle as subject to the attractions of all the others. Then the particles originally in a straight line would still in time form a spiral.

So we see that a nebula originally in the shape of a light rectilinear cloud with a condensation near the middle, like the part AB in fig. 1, would after some time exhibit a spiral like the dark part in fig. 2. The nebulæ, Herschel 1061, and H. 1337, as seen by Lord Rosse,[7] have exactly such a form. If, instead of having the nucleus in the middle, the original nebula had been denser near one extremity, like fig. 3, a simple spire like fig. 4 would

3.



4.

be the resulting spiral nebula, as we see it in H. 327, H. 1946,

[7] Prof. G. P. Bond, Director of Harvard College Observatory, kindly sent me copies of a number of Rosse's latest figures of spiral nebulæ—for which important service I here repeat my sincere thanks.

etc. A nucleus with four branches of different density and magnitude would give a spiral nebula like the beautiful object, Messier 99. If each of the two arms in figure 1 had been subdivided into two branches, H. 2084 would result.

These few remarks must be sufficient at this place. We have already deduced forms as fanciful as H. 1196, H. 131, H. 1744, and others, from simple rectilinear forms, and we hope before long to discuss this theory more at length. We here merely intended to show that the forms revealed to us by the great telescope of Lord Rosse appear to be simple mechanical consequences of the nebular theory, if applied to very large nebulæ.

§ 13. *The Law of the Planetary Distances.*

The law of the planetary distances has not as yet been discovered, though it has been most diligently sought for as the principal element of the "Harmony of the Spheres." The endeavors of Plato were in vain, and Kepler at last ascended to the truth that the present distances are not exactly the original ones. Titius, and after him Bode, came near it [§ 2, (1)]; but the *deviations* from this law remained unaccounted for, thus not giving the conformation most essential to any law.

To find the true law of the planetary distances has been our aim for nearly ten years; we hope the sequel will prove that we at length have found the solution of this problem in the following law:

The mutual distances of the planets correspond to equal intervals of time.

That this is a *fact* we will demonstrate; but *why* these intervals were equal we are not yet able fully to see—still we know that this is the simplest way in which the periodicity in the development of the nebula as found in § 10 can obtain.

Deferring a thorough discussion of the earlier attempts, (some of which are almost contemporaneous with our own solution,) to some future opportunity, we will now give the *inductive reasoning* which leads to our law above stated.

There are a few well known laws in the evolution of the nebula which embody the solution of the problem. We know that the planetary masses are insignificant as compared with the solar mass; hence we see that the orbits of the planets simply mark the equatorial band of the condensing nebula at those definite periods when the radius of the nebula had diminished to the distance of the planet. Thus we see that *the planetary distances must be functions of time.*

Or, if it be more plain, we may say that the original nebula, in contracting, left at certain intervals a few particles behind to mark the limit of the nebula at those instants. But while condensing, the uttermost particle of the nebula describes a spiral curve; and if we can find the relation between the distance a of

this particle and the time t, we need only to substitute the different intervals corresponding to the formation of the different planets in order to obtain their distances. But as the evolution is regularly periodical (§ 10), it is most probable that these intervals *are equal;* comparison with observation shows this to be the case.

But the motion of such a particle in so rare a nebula is regulated by the attraction of the whole nebula and the resistance of the ether. The first of these forces is inversely proportional to the square of the distance a, since the mass remains sensibly the same, and the particle is considered as on the equatorial *surface* of the nebula. In other words, the force of attraction on the particle is the same as the force of gravitation acting upon a planet. Resistance of the ether will necessarily follow the same law, whether a single particle or a planet be subject to it. But then our analysis[6] of the motion of a planet toward the sun is directly applicable to the motion of the superficial particle in its fall toward the center of the nebula. Formula (10) of that article shows the distance a (radius of the nebula) to be

$$a = \mathrm{A}.\varepsilon^{-2\nu t} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (35)$$

where A is the original distance (or radius of the nebula) and t the time of falling from A to a. Or, if a_t represent the distance of the planet that separated from the principal nebula at a time t *earlier* than the now nearest planet—i. e. the *age* of the planet as counted from Mercury,—the above (35) becomes

$$a_t = \beta.\gamma^t \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (36)$$

where β and γ are constants. But in the analysis leading to (35) the coëfficient ν of resistance

$$\nu = \frac{3}{8}\frac{\delta}{\varDelta\varrho} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (37)$$

has been considered constant; here we cannot do so, for though the density δ of the ether and the radius ϱ of the particle may be considered constant, the density $\varDelta$ of the particle varies very much, about inversely as the cube of the radius of the (homogeneous) nebula. If, therefore, ν' be the value of ν corresponding to the particle at the distance 30·0 of Neptune, ν'' the same at the distance 0·4 of Mercury, we have for a homogeneous nebula

$$\nu' : \nu'' = (\cdot 4)^3 : (30\cdot 0)^3 = 1 : 422000$$

nearly. If δ increases toward the center this proportion would be diminished; but still we see that ν decreases toward the interior. The formula (36) can therefore only express the principal part of the law; how (36) has to be amended in order to take

[6] On the Density, Rotation and relative Age of the Planets. This Journal, 1864, [2], xxxvii, 36.

the variation of ν into account has to be separately investigated. Before we attempt this we will compare (36) with observation.

Erecting at equal distances (§ 10) ordinates proportional to the actual planetary distances, and interpolating by connecting these points by a curve, we see that this curve has the appearance of a *logarithmic curve* (like those given on the plate appended to our former article, this Journal, 1864, vol. xxxvii); and if drawn with sufficient care, we find that the constancy of the subnormal, characteristic of the logarithmic curve, holds good in the present instance—thus proving that (36) really is applicable to the planetary distances. But it is not the exact law, for the axis in the diagram is evidently too far below the curve, or the distances are too great by a constant α, so that the diagram of the planetary distances will be expressed not by (36) but by

$$a_t = \alpha + \beta . \gamma^t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

and it remains to be seen whether this additional constant α can be accounted for by the variation of ν (37). Before we investigate this, we will see how far (38) represents observation.

We see that it is almost the same as the law of Titius, but while in the latter *t is a mere index*, it is in (38) a variable, the great independent variable of mechanics, *time* or *age!* Besides, (38) deviates from Titius in the case of Mercury. Adapting the constants of Bode to (38), it becomes

$$a_t = 4 + (1{\cdot}5) . 2^t. \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

Representing by a the actual distance, we have, for comparison with observation,

Planet.	age, t.	Distance calc. a_t.	Distance obs. a.	Difference.
Mercury, - - -	0	55	38·7	+ 16·3
Venus, - - - -	1	70	92·3	− 2·3
Earth, - - - -	2	100	100·0	0·0
Mars, - - - -	3	160	152·4	+ 7 6
Asteroids (1)–(72), - -	4	280	262·3[8]	+ 17·7
Jupiter, - - - -	5	520	520·3	− ·3
Saturn, - - -	6	1000	953·9	+ 46·1
Uranus, - - - -	7	1960	1918·2	+ 41·8
Neptune, - - -	8	3880	3003·6	+876·4

We see that the *present distances* a agree with the *original* a_t for the principal planet of both groups, for the Earth and Jupiter. Mars, Saturn, and Uranus are about $\frac{1}{20}$th of their distance too near the sun, having approached the latter so much more on account of their mass being smaller. Mercury and Neptune have even approached still more, the former because of the smallness

[8] Calculated from the table in *Smithsonian Report* for 1861, p. 218–219. We found the following interesting fact: mean distance of (1) to (31) = 2·599; of (31) to (56) = 2·679; of (57) to (72) = 2·752, showing that in general *the more distant members of the group of asteroids have been later discovered.*

of its mass, the latter on account of its high age (see this Journal, vol. xxxvii, p. 41). Before the precise influence of resistance was known, these deviations were considered sufficient cause to reject the law of Titius-Bode; but now these very deviations have become essential supports of the truth of that law.

Another and better test of our law (38), and of the constants of Bode (39), is obtained by directly solving (39) for the age t

$$t = \frac{\log (a_t - 40) - \log 15}{\log 2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

and seeing how far t is given by the series 0, 1, 2 ... We thus find

Planet.	Age.	too small.
Mercury, - - - -	imag.	
Venus, - - - - -	1·1066	—·1066
Earth, - - - -	2·0000	·0000
Mars, - - - - -	2·9056	+·094
Asteroids ①–⑫, - - -	3·890	+·11
Jupiter, - - - - -	5·0001	·000
Saturn, - - - -	5·9264	+·073
Uranus, - - - - -	6·9683	+·031
Neptune, - - - -	7·6230	+·377

From this table we see that the age of the planets above that of Mercury is as the series of natural numbers, *the deviations not only being but small*, but just such as influence of the mass would make them. This may be easily proved by the formula contained in the article on the age of the planets before referred to.

If the present age of Mercury be m, then the age of the interior planets will be to that of the exterior ones as $m + \frac{6}{4}$ is to $m + \frac{26}{4}$, or as $2m + 3$ to $2m + 13$. We found this ratio as 1 to 3 (this Journal, vol. xxxvii, p. 43); if true it would follow that $m = 1$, or the *total age of any planet would be* $t + 1$, *the unit being the age of Mercury.*

After having seen that (38), the modified form of (36), is applicable to the planetary distances, we will demonstrate that this modification is consistent with the signification of t, the time.

If the resistance R be proportional to the velocity v, or

$$\mathrm{R} = \nu v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

we have the tangential force (this Journal, vol. xxxvii, p. 40)

$$\frac{1}{r} \frac{d\left(r^2 \frac{d\theta}{dt}\right)}{dt} = -\mathrm{R} \cos \eta = -\nu r \frac{d\theta}{dt}, \quad . \quad . \quad (42)$$

where r is the radius vector, and θ the anomaly; but Kepler's second law gives

$$r^2 \frac{d\theta}{dt} = c, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

so that (43) becomes

$$\frac{1}{r}\left[\frac{dc}{dt}+\nu c\right]=o, \quad \ldots\ldots\quad (44)$$

giving *for* ν *constant*,

$$c=\mathrm{C}.e^{-\nu t}, \quad \ldots\ldots\quad (45)$$

or, since by Kepler's third law, $c^2=a\mu$,

$$a=\frac{\mathrm{C}^2}{\mu}e^{-2\nu t}, \quad \ldots\ldots\quad (46)$$

all of which formulæ are demonstrated in our article referred to above.

Instead of solving the problem directly, we may indirectly try to find how ν must vary that (36) may become (38), i. e. to add a constant term to (46). In other words, C instead of being constant must be considered a function of t, i. e. (44) must be

$$\frac{1}{r}\left[\frac{de}{dt}+\nu c\right]=\varphi(t), \quad \ldots\ldots\quad (47)$$

so that the resistance now becomes, see (42),

$$\mathrm{R}=\nu v-\frac{\varphi(t)}{\cos\eta}=\nu v-\frac{ds}{d\theta}.\frac{\varphi(t)}{r}, \quad \ldots\ldots\quad (48)$$

instead of (41), where $\cos\eta=\frac{rd\theta}{ds}$, and ds is the element of the orbit. The function $\varphi(t)$ can now, by the method of the variation of the arbitrary constants, be so determined that (46) or (36) coincides with (38). Since r is a function of t, we may make

$$f(t)=r\,\varphi(t), \quad \ldots\ldots\quad (49)$$

hence (47) becomes

$$\frac{dc}{dt}+\nu c=f(t). \quad \ldots\ldots\quad (50)$$

Taking the complete differential of (45), i. e. also considering C variable, substituting in (50) and reducing by (45), we obtain for the determination of C,

$$e^{-\nu t}.\frac{d\mathrm{C}}{dt}=f(t). \quad \ldots\ldots\quad (51)$$

This gives, by making K an arbitrary constant,

$$\mathrm{C}=\mathrm{K}+\int e^{\nu t}.f(t).dt, \quad \ldots\ldots\quad (52)$$

which, substituted in (46), gives,

$$a=\frac{e^{-2\nu t}}{\mu}\left[\mathrm{K}+\int e^{\nu t}.f(t).dt\right]^2. \quad \ldots\quad (53)$$

This should be identical with (38), i. e. (remembering that t here is counted from the most distant, in (38) from the nearest planet and that γ in (36) is $e^{-2\nu}$ in (35))

$$a=\alpha+\beta.e^{-2\nu t}. \quad \ldots\ldots\quad (54)$$

Equating (53) and (54), and solving for $f(t)$, we find,

$$f(t) = \nu \frac{\alpha \sqrt{\mu}}{\sqrt{\alpha + \beta e^{-2\nu t}}}, \quad . \quad . \quad . \quad . \quad (55)$$

or, by (54), $$f(t) = \nu \alpha \sqrt{\frac{\mu}{a}}. \quad . \quad . \quad . \quad . \quad . \quad (56)$$

But Kepler's third law gives $\mu = a . v^2$ (this Journal, vol. xxxvii, p. 38, note); hence

$$f(t) = \nu \alpha v, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

consequently, by (49), r and a being now the same again,

$$\varphi(t) = \nu \alpha \frac{v}{a}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

or (48), $\cos \eta$ being almost equal to one, the orbit being nearly circular, $$\mathrm{R} = \nu v \left(1 - \frac{v}{a}\right). \quad . \quad . \quad . \quad . \quad . \quad (59)$$

Thus we see that (36) becomes (38) if the resistance R, instead of being simply proportional to the velocity (41), is varying according to (59), which may be comprehended in (41) by taking the factor ν to decrease from $\nu \; (a = \infty)$ to $0 \; (a = \alpha)$ according to

$$\nu' = \nu \left(1 - \frac{\alpha}{a}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

This variation of the coefficient of resistance is conformable to (37), since $\varDelta$, according to (16) (then δ), increases as a decreases.

The law $$a_t = \alpha + \beta . e^{-2\nu t} = \alpha + \beta . \gamma'$$
is, therefore, but an amplification of

$$a_t = \beta . e^{-2\nu t} = \beta . \gamma^t :$$

in the latter the coefficient of resistance *is constant*, in the former it *varies according to* (60). As now (60) is real, (54) or what is the same (38) is the real law of the planetary distances, t continuing to represent the age, and not, as in Bode's law, a mere index. *And as now finally* (38), *applied to the actual distances, gives values for t that are very nearly as the natural numbers, our law, announced above, holds true, that the planetary distances correspond to equal intervals of time ;* or *the consecutive planets were abandoned at equal intervals of time.*

There remain yet two remarkable consequences to be drawn from this exponential law of the planetary distances. If in (38) *it is sufficiently great* (i. e. the corresponding planet far from the center) to make the first term insignificant as compared to the second, we have approximatively

$$a = \beta . \gamma^t,$$

hence $$a^{t+1} = \beta . \gamma^{t+1},$$

consequently $$\frac{a_{t+1}}{a_t}=\gamma, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

or *for the most distant planets their distances approach to a simple geometrical series whose quotient is the base* γ. But this law will again in part be interfered with on account of the action of resistance on the completed system, which, on account of the higher age, has drawn the most distant planets comparatively nearer to the sun than the less distant ones, so as to *diminish* the above quotient γ.

Again, if t is sufficiently small—or the planet sufficiently near the center—the exponential series contained in (38) is highly convergent, so that perhaps the approximation may be sufficient if only the term of the first order is taken, so that (38) becomes, A and B representing constants,

$$a_t = \mathrm{A}+\mathrm{B}.t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

hence $$a_{t+1} = \mathrm{A}+\mathrm{B}(t+1),$$

$$a_{t+2} = \mathrm{A}+\mathrm{B}.(t+2), \text{ etc.}$$

or, $$a_{t+1} - a_t = a_{t+2} - a_{t+1} = \mathrm{B}, \quad . \quad . \quad (63)$$

i. e. *the innermost planets have a tendency to become equidistant.*

Both of these consequences are very plainly marked in the solar system, especially in the lunar, but also in the planetary orbs. For, as regards (61), we have for the distances of

Saturn	to	Jupiter	as	1·85	to	1.
Uranus	"	Saturn	"	2·01	"	1.
Neptune	"	Uranus	"	1·57	"	1.

For Saturn–Jupiter this proportion is still less than $\gamma = 2$—also because Jupiter both on account of its age and mass has fallen less toward the sun than Saturn; but for Uranus–Saturn the ratio is almost equal to $\gamma = 2$, while for Neptune–Uranus it is less again, on account of the higher age of the first.

The second circumstance, expressed in (63), seems to be exemplified in the orbits of Mercury, Venus, Earth, the three planets that are nearest to the sun, or for which t is the smallest. Their distances are

	Distance.	Difference.
Mars, - - - -	152·4	
		52·4
Earth, - - - -	100·0	
		26·7
Venus, - - - -	72·3	
		33·6
Mercury, - - -	38·7	

We see how Mars, Earth and Venus follow Bode's law exactly, for one-half of 52·4 is 26·2, or very nearly 26·7—but the distance between Venus and Mercury is 33·6 instead of $\frac{1}{2}$ of 26·7 or 13·4. This differencc might be considered as a consequence of (63); but we know that it is principally due to the small mass of Mercury.

§ 14. *The Lunar Distances.*

As Kepler's third law was deduced from the planetary orbits alone, so was the law of Titius. But it was shown to be a consequence of the law of universal gravitation, and therefore itself universal and applicable to any system—hence, also to the lunar systems. Now the law of Titius, as modified above, has been found to be identical with the equality of the intervals of time in the history of any system. Therefore, also, this law (38) must apply to the lunar systems. This we now will show.

A. *The Lunar System of Jupiter.*

The *Jovial World* is the youngest of those great lunar systems that adorn the exterior planets. (This Journal, xxxvii, p. 45.) Therefore, it is the most regular yet of any, and our law (38) must very closely harmonize with the actual distances of Jupiter's moons. It is easily found that $\gamma=2$, again, as for the planetary distances; and that $\alpha=4$ and $\beta=3$ radii of Jupiter. Thus (38) is for the Jovial World,

$$a_t=4+3\times 2^t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

	t.	Distance. Calculated.	Distance. Observed.	Fall.
Moon I.	0	7	6·049	·951
" II.	1	10	9·623	·377
" III.	2	16	15·350	·650
" IV.	3	28	26·998	1·002

The "fall" of a moon is the distance it has fallen toward the planet in virtue of the resisting ether. That this fall corresponds to the age, mass and density of the different moons has been shown in our previous article. (This Journal, xxxvii, 45.)

The calculation of t from the observed distances gives for the 2d, 3d and 4th, respectively, ·907, 1·92, and 2·94, which only deviate by ·09, ·08 and ·06 from the theoretical values 1, 2 and 3; and all values being *too small* shows that these moons are correspondingly nearer the primary, having approached so much on account of the etherial resistance.

B. *The lunar world of Saturn*

is next in age, hence not quite so regular as that of Jupiter. We find that (38) represents the distances of the eight moons if the constants are

$$a_t=4+0{\cdot}35\times 2^t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

as will be seen from the following table:

Moon.	Distance. Calculated.	Observed.	Difference.
I. Mimas,	4·35	3·4	+·95
II. Enceladus,	4·70	4·3	+·40
III. Tethys,	5·4	5·4	·0
IV. Dione,	6·8	6·8	·0
V. Rhea,	9·6	9·6	·0
VI. Titan,	15·2	22.2	−7·0
VII. Hyperion,	26·4	28·0 ?	−1·6 ?
VIII. Japetus,	48·8	64·0	−15·2

Excepting for a moment the 6th and 8th moon, we see but small differences; Mimas and Enceladus being too near Saturn, appear to have but very small mass, which conclusion is strengthened by the fact that it required Herschel's great telescope to discover them (1789). The next three almost exactly harmonize with this law; they are, therefore, not only larger than the first two, but also much alike. They were discovered by Cassini, first the fifth (Rhea) in 1672, and later (1684) Tethys and Dione. As the latter were discovered by the same observer, the difference in date is, perhaps, alone due to the greater nearness to the disk of the primary. Hyperion is even lower than any, and, therefore, smaller than even the interior ones. This is confirmed by its discovery, which was not made till 1848, by Bond and Lassell. But the sixth, Titan, and the eighth, Japetus, are much farther distant than (65) gives; thus proving them to have much more considerable mass (or rather v (37) is less, which in general will be the case if the mass is greater). This is fully confirmed by the date of discovery: Titan being the first discovered of all, (by Huyghens, 1655), and Japetus the second, (by Cassini, 1671). These estimates of the masses are further corroborated by Humboldt,[1] who calls Titan "the largest of all known secondary planets." Compare another theoretical estimate, (this Journal, xxxvii, 46), leading to the same results.

C. *The lunar system of Uranus*

is exceedingly important on account of the plane and direction of its motions. We have tried to show that this very position affords one of the most conclusive confirmations of the nebular theory. (This Journal, xxxvii, 50.) Here we will consider the arrangement of the individual members of the system.

We know it to be the oldest, because it is the most distant system of which we have definite knowledge. The original distances and the original harmony of these distances is therefore here most deranged. We cannot even with any degree of certainty consider the moons to be now in the same order of suc-

[1] *Cosmos*, i, Harper's edit., p. 95.

cession as at first. At the same time, observation has as yet hardly determined the number, much less the exact distance of the different moons. Therefore, we give the following more for the sake of completeness than with the view of adding any important confirmations of our law.

We have seen that the nearest luminaries may be equi-distant, and that the farthest may succeed at distances that form a geometrical progression--see (61) and (63). If the distances, as given by Herschel, and the times of revolution, as given by Lassell,[2] are exact, we may represent the distance of the first six moons by

$$a_t = 7{\cdot}5 + 3t, \quad . \ . \ . \ . \ . \ . \ . \ . \quad (66)$$

corresponding to (62), and the distance of the sixth, seventh and eighth by

$$a_t = a_6 \, . \, 2^{t-6}, \quad . \ . \ . \ . \ . \ . \ . \ . \quad (67)$$

corresponding to (61).

	Distance. Calculated.	Observed.	Difference.
Moon I	$=7{\cdot}5+0\times 3 = 7{\cdot}5$	7·5	·0
II	$=7{\cdot}5+1\times 3 =10{\cdot}5$	10·5	·0
III	$=7{\cdot}5+2\times 3 =13{\cdot}5$	13·1	+ ·4
IV	$=7{\cdot}5+3\times 3 =16{\cdot}5$	17·0	− ·5
V	$=7{\cdot}5+4\times 3 =19{\cdot}5$	19·8	− ·3
VI	$=7{\cdot}5+5\times 3 =22{\cdot}5$	22·7	− ·2
VII	$= 2\times 22{\cdot}5=45{\cdot}0$	45·5	− ·5
VIII	$= 4\times 22{\cdot}5=90{\cdot}0$	91·0	−1·0

If these observed distances really are correct, then this remarkable discontinuity will enable us to determine the lunar masses long before observation can ascertain them.

D. *Conclusion.*

The lunar system of the *Earth*, consisting of but *one* moon and that of *Neptune*, which comprehends one or two, cannot, or do not afford any chance to test our law. But we have seen that the systems of Jupiter and Saturn fully confirm our law (38), if due regard is had to the individual mass and volume—or density and radius—of the several moons. Even the system of Uranus, as far as known, does not deviate from it except in so far as it offers the two extreme limits of the law, probably on account of the high age and a close similarity between the masses of the first six moons.

Therefore we may say that as far as observation on the lunar systems goes it is embodied in our law (38), or *in every lunar system the consecutive moons were formed at equal intervals of time.*

[2] See Schweigger in Astronomische Nachrichten, No. 832, Beilage.

§ 15. *The incommensurability of the periodic times.*

By the third law of Kepler we have, if T_t and $T_{t'}$ are the periodic times of two planets,

$$\frac{T_{t'}}{T_t}=\left(\frac{a_{t'}}{a_t}\right)^{\frac{3}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

or by (38),

$$\frac{T_{t'}}{T_t}=\left[\frac{\alpha+\beta\gamma^{t'}}{\alpha+\beta\gamma^{t}}\right]^{\frac{3}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

which expression will not generally make $T_{t'}$ and T_t commensurable. Thus we see that our law accounts for another important condition of stability of the system, (see § 3, 1).

But as the distances are continually decreasing, and at different rates, (this Journal, xxxvii, 41, gives the numerical values of these rates), we perceive that *in time such commensurability may take place* between any two planets.[3] Such is actually the case between Jupiter and Saturn, as discovered by Laplace.

The distances were (see § 13) for Jupiter $a_5 = 520$, for Saturn $a_6 = 1000$, giving for (68) the continued fraction $2(1, 1, 2\ .\ .\ )$ having the approximations,

$$\frac{2}{1}, \ \frac{3}{1}, \ \frac{5}{2}, \ \frac{13}{5} \quad . \quad . \quad .$$

or $T_6 : T_5$ approached originally to 5 : 2; now it is very nearly so.

For Venus and the Earth the original distances 70 and 100 give the approximations,

$$\frac{1}{1}, \ \frac{2}{1}, \ \frac{5}{2}, \ \frac{12}{7}, \ \frac{29}{17} \quad . \quad . \quad .$$

whilst Airy has found the commensurability 13 : 8 or nearly our 29 : 17 [13 : 8=29 : 17·8].

In the *lunar systems* such commensurability is common; and it is for the satellites of Jupiter that Laplace demonstrated[4] the great proposition, *if such commensurability exists but approximatively it will become exact in time.*

Having seen that the change in distance produced by resistance will make the ratio approach commensurability, it therefore, as we stated before, will become rigorously so.

From (68) we find easily that the ratio will be 2 if the distances are in the ratio of $2^{\frac{3}{2}} : 1$, or (by continued fractions) as the approximative fractions,

$$\frac{1}{1}, \ \frac{2}{1}, \ \frac{3}{2}, \ \frac{8}{5}, \ \frac{19}{12}, \ \frac{27}{17}, \quad . \quad . \quad .$$

[3] Grant (*History of Physical Astronomy*, London, 1852, p. 93,) states that the libration of the jovial moons is "independent of the effects of a resisting medium," meaning that it will be preserved notwithstanding such medium. This is probably a mistake, for it would depend upon the relative magnitude of the resistance and the perturbation.

[4] Méc. Cél., vol. viii, Ch. vi, § 15. We express the proposition in more general terms.

For Jupiter's satellites we have $a_2=16$, $a_1=10$, or $a_2 : a_1=8:5$; and $a_1 : a_0=10:7=8:5{\cdot}6$, therefore we find the periodic time of the second moon twice that of the first, $T_1=2T_0$, and the periodic time of the third twice that of the second, $T_2=2T_1$; hence Laplace's famous relation between the mean motions,

$$n_0+2n_2=3n_1.$$

In the system of Saturn similar relations obtain. At first we had (see § 14, B) for the distance of Tethys (III) and Mimas (1) the ratio 5·4 : 4·4=8 : 6·6, while the duplication of the periodic times requires the ratio 8 : 5. But Mimas has approached Saturn the most, and thus this proportion (now 5·4 : 3·4=8 : 5·04) has been brought about.

For the fourth and second we had originally, Dione : Enceladus=6·8 : 4·7=8 : 5·5, or likewise sufficiently near 8 : 5 that the duplication of the periodic time should become almost rigorous.[5]

The lunar world of Uranus is particularly noted for such duplications, from the fact that Schweigger, as early as 1814, on such grounds predicted the existence and gave the orbits of the two innermost moons of Uranus, which were discovered by Lassell in 1851. The coincidence is very remarkable, as will be seen from the following:[6]

	Schweigger, 1814.	Lassell, 1851.
Uranus, I moon, - - -	2·1767 days,	2·5117 days.
II - -	4·3534 "	4·1445 "

and the IV (or II of Herschel) having a period of 8·7068 days approximates to the further duplication of the periodic times. Also the period of III is about half the IV period, the former being 5·8926 days, the latter 10·9611.

Taking only the first two decimals we find by means of continued fractions the following approximations:

$$\text{II to I or } \frac{441}{251}=\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{28}{17}, \frac{33}{20}, \text{ etc.}$$

$$\text{IV `` II ``} \quad \frac{871}{441}=\frac{2}{1}, \frac{19}{9}, \frac{21}{10}, \frac{40}{19}, \frac{61}{29}, \text{ etc.}$$

$$\text{V `` III ``} \quad \frac{1096}{589}=\frac{1}{1}, \frac{2}{1}, \frac{13}{7}, \frac{93}{50}, \frac{199}{107}, \text{ etc.}$$

thus proving that only the fourth (Herschel II) and second (Lassell II) have periodic times nearly in the ratio of 2 to 1.

The other instances adduced by Schweigger, and especially the first, do not seem to have any claim to be considered as real duplications. Still it is evident that the configuration of the Uranian-system is such as approaches to simple ratios between the periodic times; and if the perturbating force arising here-

[5] Herschel, *Outlines of Astronomy*, § 550.

[6] Schweigger; Ueber die Auffindung der ersten Uranustrabanten durch Lassell. Astronomische Nachrichten. 1852, No. 832.

from is greater than the effect of resistance, these ratios and the corresponding configuration would become permanent. It is not improbable that an analysis of the lunar system of Herschel's planet will throw much light on the future configuration of the solar world by ascertaining the exact relation between perturbation in commensurable revolutions brought about by resistance and the continued influence of the latter force on such commensurable motions.

Though this latter question cannot at present be fully answered, we have proved in this paragraph that not only the general incommensurability of the periodic times ensuring the stability of the system, but also the deviations therefrom are accounted for by our law (38).

§ 16. *History of the Solar System.*

Believing that we have, in the preceding pages, brought forth some further arguments in favor of the nebular hypothesis, we may be permitted in a very few words to sketch the grand history of the material universe as it is seen in the light of this theory. The philosophers of old called Man a Microcosmos—we compare the Universe, the Macrocosmos, to man, thereby intimating that as Man has a parentage, growth and decay, i. e., a *history*, so has the Macrocosmos.

The history of the *material* world may be divided into four periods or ages, corresponding to those given in a note to § 6. (Compare Guyot's views in Dana's Geology—chapter on "Cosmogony").

In the beginning *God created the heavens and the earth.* And the earth was *without form and void,* and *darkness* was upon the face of the deep. And the spirit of God moved upon the face of the waters. (*Genesis*, I, 1, 2).

The material universe was created *not* in its present form, but without form; it was *void* and *dark*; but the spirit of God pervaded it, and planned it such that his All-Foresight, or Providence, might also be manifest in the material world. *This* is really *the Creation*—it is merely stated, not described, for it is inconceivable to mortal understanding. It is too awful, our mind is lost in reflecting thereon; hence the divine writer merely mentions it at the beginning, and, to give fullness to his picture and adapt himself to our understanding, describes the first three great ages as real creative acts, though mere consequences of the unfathomable word given in the *first* verse of Genesis. We believe that the first five verses of Genesis have never before been fully understood in their deepest sense. We shall in the sequel keep constantly before our eyes both this, the *revealed History of the Cosmos*, and *science*, deduced from the revelation we have in the present form of nature.

Since Genesis merely states that the universe (i. e. heaven and earth) was *formless*, *void* of any organized being and *dark*, it is science alone that can give us any idea of the constitution of the universe as it came from the hands of the Creator. But as science is progressive, our ideas of the primeval condition of the Cosmos must progress correspondingly, or rather with advancing science our eye pierces farther and farther back into the dark past, approaches more and more to the mysterious and almighty "Fiat." As these approaching steps represent greater and greater series of ages, we infer that the *Fiat* lies infinitely far behind us, and can never be reached by human thought. We experience in regard to the age of Cosmos by penetrating farther and farther into the dark past with our spiritual eye, the same that we feel in piercing, by means of more and more powerful telescopes, farther and farther into the world-filled abyss of space. Here, if looking through a giant telescope we find ourselves surrounded by a boundless space filled with the wonders of the Creator; and if ardently searching in the existing documents of nature for records of her past, we behold infinity also here, the infinity of time, *eternity*, teeming with wonders no less astounding. The beautiful poem of Schiller, "*die Grösse der Welt*," is true both as to the extension and the duration of the World.

The ancients most frequently thought that the world left the hands of the Creator in the shape it now is. Even Newton himself was unable to see farther back. To him the Creator was but a tinker, forming his wheeling globes and wheeling them around their axis, putting them one by one and one by one to their very place in his clockwork—to him an unorganized machine to run on and on forever in the same shape. But Huyghens, and Newton himself, by discovering the generic cause of the figure of the earth aimed the first blow at this base idea, which nevertheless has found its advocates even to the present hour, especially among theologians. The corner-stone being broken out of the system it has been crumbling down. Geology has restored the lost history of the earth, and the nebular theory has traced this earth to the sun as her mother. Thus creation was now identical with the productions of the rotating mass of matter, i. e., of the chemical elements.

We have attempted to show that both rotation and the elements come from the forces wherewith the ONE matter (Urstoff) was endowed (see § 6). It is highly interesting to see how the first verse of Genesis has been understood by scientific men. It will at the same time more clearly set forth what we implied above when saying that science is approaching to the true original condition of Cosmos by making steps representing longer and longer periods of time.

"In the beginning God created the heaven and the earth" means according to

Newton, 1686: a direct, immediate creation of every globe as it is now.

Huyghens, Hutton, and modern Geologists: a direct creation of the heavenly globes as *fiery masses*, circulating in the system as they do now.

Kant, 1755, Laplace (later): a direct creation of a rotating mass of chemical elements; giving rise to the planetary system.

We, in 1854, conceived this rotating mass of elements to be the product of a created nebula consisting of but *one single element.*

We will now contemplate the different ages manifest in the development of this Urstoff.

First Day or Age.—The atoms of "Urstoff" combine—light (and heat) and the *chemical elements* result. The mere production of light would not entitle it to be considered one of the days of creation; but light is by the divine writer taken as a type to represent itself, and the less obvious, though much more important, chemical elements. It was not so much the light as the formation of the elements, the basis of modern physical science, which characterized the first day. We think that a rotation was also produced hereby. (This Journal, 1864, vol. xxxvii, p. 52.)

Second Day or Age.—Formation of the planetary orbs with their satellites.—The nebula developed itself into a great number of similar planetary nebulæ, which again gave birth to similar lunar nebulæ. Thus we see here the simplest kind of "life," reproduction by division, as exhibited by many plants, and even animals, which to distinguish them as such from inanimate matter, have another mode of reproduction *besides.* The planets represent the children, the moons the grandchildren of the sun.

Third Day or Age.—The fiery balls resulting from this subdivision cool down and are shaped, as Geology has ascertained in relation to our own earth.

The Fourth Age of the *inorganic era* is the present. We have shown that the further characteristic of life, namely, death, is not restricted to the organic but is participated in by inorganic nature (this Journal, [2], xxxvii, 56). As every breath of our lungs is a differential of decay—so every rotation of the earth giving us the enjoyments of another day, and every revolution charming us with the succession of the seasons, brings our own mother earth nearer to her grave.[1]

[1] We beg the scientific reader's pardon for these paragraphs, which do not belong to this place. But we felt it urgent to say at least this much, as some, even to-day, are apt to base the cry of "heretic," "infidel," etc., on any such deviation from the beaten path in their dogmas. The nebular hypothesis has richly participated in the abuse heaped in its day on the Copernican system, and on some leading doctrines of geology. Even yesterday, I found, in one of the leading religious quarterlies, Laplace called an "atheistic dreamer"! We wrote this paragraph as a protest against such imputations.

§ 17. *Conclusion.*

The principal results arrived at in this paper are

1st, A simple mechanical *theory of spiral nebulæ.*

2d, A more accurate determination of the orbits; and above all,

3d, *The discovery of the true law of the planetary and lunar distances.*

4th, The determination of the periodic times as a function of the distances—or borrowing this third Law of Kepler from the theory of gravitation, we have therein almost a theoretical demonstration of *the equality of the intervals.*

As (38) what we have repeatedly called "our law" is very much like Bode's, or rather Titius's, law, we apprehend that the propriety of thus naming (38) will be doubted. To set this point in clear light we refer to a similar, though undoubtedly grander, case in the history of science.

The law of Titius was exclusively derived from observation. It is empirical, as is the third law of Kepler. It is, moreover, not exact, neither in its general form nor in its numerical results. But neither is the famous law of Kepler exact, though, on account of the different circumstances connected herewith, this latter law agrees better with the numerical data of observation than Titius's law.

Newton discovered the true form of Kepler's law by deducing it from a higher law, that of universal gravitation. Instead of Kepler's form, C being the same constant for all planets,

$$\frac{T_2}{a^3} = C, \quad . \quad . \quad . \quad . \quad . \qquad (70)$$

Newton found that the true law is

$$\frac{T^2}{a^3} = \frac{\mu}{4\pi^2}(M + m), \quad . \quad . \quad . \qquad (71)$$

μ being the constant of gravitation, hence the same for all planets; hence,

$$C \propto (M + m). \quad . \quad . \quad . \qquad (72)$$

That is, Kepler's constant C is proportional to the sum of the mass M of the sun and the mass m of the planet. By farther analysis it is found that C even is dependent on *all* the masses and distances in the system.

So also in our case. We have given the true expression of Titius's law by extending it to Mercury and have accounted for the deviations of nature from the law, by demonstrating that it is a necessary consequence of the higher law, viz: *the intervals between the abandonment of the different orbs of the same system are equal* (see § 13). *Now this is what we claim as our law.* As Newton deduced and corrected Kepler's law by his law of *equal gravitation*, so we have deduced and corrected the law of Titius by our law of *equal intervals.*

We referred to (38) as our "law" because it is a *consequence* of our law, and certainly our formula; we did not intend to obliterate the merit of Titius, as will be seen wherever we have mentioned his name.

There is yet another circumstance which makes our demonstration of the law of planetary distances so important. It is the touchstone of the nebular theory; for as this ascribes the formation of the planets to the slow descent of cosmical matter to its center, it has to be proved that such descent will give exactly the actual system. Already Plato held that[7] "the motion of the planets is such as if they had been all created by God in some region very remote from our system, and let fall from thence toward the sun, their falling motion being turned aside into a transverse one whenever they arrived at their several orbits." Galileo was the first who subjected this "concetto platonico" as he calls it to a numerical calculation based upon the laws of falling bodies as discovered by him. He finds an admirable harmony between his calculations and the actual velocities and distances as they were known at his time.[8] Next after him, Newton took the matter in hand, and in his third letter to Dr. Bentley he gives as his result, that it is impossible to account for the configuration of the system in the manner of Plato and Galileo. This result is based upon his assumption of a vacuum. By taking the influence of a resisting medium into account, we have proved that the Platonic idea as embodied in the nebular hypothesis *does* lead to the present configuration of the solar world. We make these remarks to show that the idea we advocate is old and venerable; we hope, at some other time, to give the highly interesting history of the law of planetary distances, including the application of the Phyllotaxis, (Pierce, Agassiz,) the radius of gyration, (Kirkwood,) the regular polyhedra, (Kepler, Plato,) etc.

How grand and beautiful is the harmony of the planetary world! What an admirable *unity of plan* is manifested therein! As *now* the planets are slowly sinking to the sun, so they have *always* been sinking since the moment of their creation as a nebulous mass; the same motion that now brings them nearer to their death has caused their formation, has brought them to life! And how sublime is the plan of creation! To call forth the harmonious system of the solar world with all its multiform aspects and dependencies fit to support life throughout almost endless ages—nothing but a collection of matter endowed with its

[7] Brewster, *Life of Newton*, Ch. 16.

[8] *Dialogo intorno ai due massimi Sistemi del Mondo, Tolemeico e Copernicano.* Gjornata I, (ed. *Opere*, Firenze, 1842. Vol. i, p. 34–35.) He finds: le grandezze dei cerchj, e le velocità dei moti s'accostano tanto prossimamente a quel che ne danno i computi, che è cosa maravigliosa.

molecular forces was placed in a little spot of the house that contains many mansions besides. This matter slowly collected together. In thus following the force of attraction planted in it by eternal love, the whole great life of the solar world was awaked; and as the pulsation of the heart in man indicates the fleeting moments of his life, so the pulsations of that great whole, succeeding each other at equal intervals, gave each one birth to a new world to mark the historic epochs of the Universe by its position and to roll on for ages, a revelation of the Great Author, until, always following the same attractive force, it in death finds rest at the bosom of the planet-mother, the sun. And then—this grand system remains as a mere lump, a *Cosmic Fossil*, suspended in space, where perchance some higher being may meet with it, touch it, investigate it, and construct its whole past history, as the geologist in our days studies the history of a fossil shell!

Iowa City, Iowa, July—November, 1864.

ERRATA.

P. 9, line 13 from bottom, for "$t=1, 2, 3$," read "$t=0, 1, 2, 3$."

P. 16, foot-note 3, insert "om" before Stoffernes.

P. 26, line 18 from bottom, in table, for "92·3," read "72·3."

p. 28, in formula (47), for $\frac{de}{dt}$ read $\frac{dc}{dt}$.

p. 29, l. 19 from bottom, for γ' read γt.